$$\Delta x = \frac{b-a}{n} \qquad f(x) = x^2 - 2x + 1 \qquad x_k = a + k\Delta x$$

$$= \frac{1-0}{n} \qquad\qquad = 0 + k\left(\frac{1}{n}\right)$$

$$= \frac{1}{n} \qquad\qquad = \frac{k}{n}$$

$$\int_0^1 x^2 - 2x + 1\, dx = \lim_{n\to\infty} \left(\sum_{k=1}^{n} \left(\left(\frac{k}{n}\right)^2 - 2\left(\frac{k}{n}\right) + 1 \right) \right) \Delta x$$

$$= \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{k^2}{n^2} - \sum_{k=1}^{n} \frac{2k}{n} + \sum_{k=1}^{n} 1 \right) \frac{1}{n}$$

$$= \lim_{n\to\infty} \left(\frac{1}{n^2} \sum_{k=1}^{n} k^2 - \frac{2}{n} \sum_{k=1}^{n} k + n \right) \frac{1}{n}$$

$$\frac{5}{5}$$

$$= \lim_{n\to\infty} \left(\frac{1}{n^2} \left(\frac{n(n+1)(2n+1)}{6} \right) - \frac{2}{n} \left(\frac{n(n+1)}{2} \right) + n \right) \frac{1}{n}$$

$$= \lim_{n\to\infty} \left(\frac{1}{n} \left(\frac{(n+1)(2n+1)}{6} \right) - (n+1) + n \right) \frac{1}{n}$$

$$= \lim_{n\to\infty} \left(\frac{2n^2 + 3n + 1}{6n} - \cancel{n} - 1 + \cancel{n} \right) \frac{1}{n}$$

$$= \lim_{n\to\infty} \left(\frac{2n^2}{6n} + \frac{3n}{6n} + \frac{1}{6n} - 1 \right) \frac{1}{n}$$

$$= \lim_{n\to\infty} \left(\frac{n}{3} + \frac{1}{3} + \frac{1}{6n} - 1 \right) \frac{1}{n}$$

$$= \lim_{n\to\infty} \left(\frac{n}{3} - \frac{2}{3} + \frac{1}{6n} \right) \frac{1}{n}$$

Awesome!

$$= \lim_{n\to\infty} \frac{n}{3n} - \frac{2}{3n} + \frac{1}{6n^2}$$

$$\int_0^1 x^2 - 2x + 1\, dx = \frac{1}{3} - 0 + 0 = \frac{1}{3}$$

1 $\int_0^1 x^2-2x+1\,dx = \frac{1}{3}$

Andrew Shaffer
Colin Evans

$D = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$

$x_k = a + k\Delta x$

$\Delta x = \frac{b-a}{n}$

$x_k = 0 + \frac{k}{n}$

$\Delta x = \frac{1-0}{n} = \frac{1}{n}$ ✓

$\lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(\frac{k}{n}\right)^2 - 2\left(\frac{k}{n}\right) + 1\right]\frac{1}{n}$

$= \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{k^2}{n^2} - \frac{2k}{n} + 1$ ✓

$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{n^2}\sum_{k=1}^{n} k^2 - \frac{2}{n}\sum_{k=1}^{n=1} k + \sum_{k=1}^{n} 1\right]$

$= \lim_{n\to\infty} \left[\frac{1}{n^3}\sum_{k=1}^{n} k^2 - \frac{2}{n^2}\sum_{k=1}^{n} k + \frac{1}{n}\sum_{k=1}^{n} 1\right]$

$= \lim_{n\to\infty} \left[\frac{1}{n^{\cancel{3}}}\left(\frac{\cancel{n}(n+1)(2n+1)}{6}\right) - \frac{2}{n^{\cancel{2}}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right) + \frac{1}{\cancel{n}}\cdot \cancel{n}\right]$

$= \lim_{n\to\infty} \frac{1}{n^2}\left(\frac{(n+1)(2n+1)}{6}\right) - \frac{\cancel{2}}{n}\left(\frac{n+1}{\cancel{2}}\right) + 1$

$= \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2} - \frac{n+1}{n} + 1$

$= \lim_{n\to\infty} \frac{2n^2+3n+1}{6n^2} - \frac{n+1}{n} + 1$ ✓

$= \lim_{n\to\infty} \cancel{\frac{2n^2}{6n^2}}^{1/3} + \cancel{\frac{3n}{6n^2}}^{0} + \cancel{\frac{1}{6n^2}}^{0} - \cancel{\frac{n}{n}}^{1} + \cancel{\frac{1}{n}}^{0} + 1$ ✓

$= 1/3 + 0 + 0 - 1 + 1 \;\boxed{= 1/3}$

Check

$\int_0^1 x^2 - 2x+1\,dx = 1/3$

$= \frac{x^3}{3} - x^2 + x \Big|_0^1$

$= \left[\frac{1}{3} - \cancel{1} + \cancel{1}\right] - \left[\cancel{\frac{0}{3} - 0 + 0}^{0}\right]$

$= \boxed{1/3}$

$\frac{5}{5}$

Excellent!